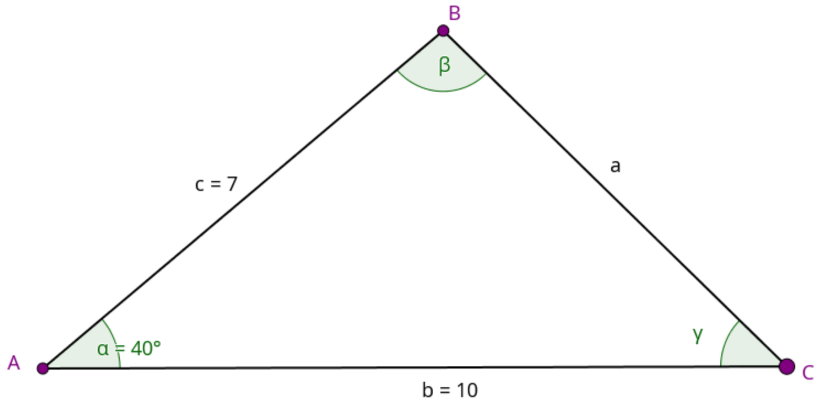
Du théorème du sinus au théorème du cosinus



$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$

$\beta = ?$; $\gamma = ?$; $a = ?$ $\quad \frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$

$$\frac{b}{\sin(180° - (\alpha + \gamma))} = \frac{c}{\sin(\gamma)}$$

$$\frac{b}{\sin(\alpha + \gamma)} = \frac{c}{\sin(\gamma)}$$

$$b \cdot \sin(\gamma) = c \cdot \sin(\alpha + \gamma) = c \cdot (\sin(\alpha) \cdot \cos(\gamma) + \cos(\alpha) \cdot \sin(\gamma))$$

$$\frac{b}{c} \cdot \sin(\gamma) = \sin(\alpha) \cdot \cos(\gamma) + \cos(\alpha) \cdot \sin(\gamma)$$

$$\sin(\gamma) \cdot \left(\frac{b}{c} - \cos(\alpha)\right) = \sin(\alpha) \cdot \cos(\gamma) \qquad \left(\tan(\gamma) = \frac{\sin(\alpha)}{\frac{b}{c} - \cos(\alpha)} = \frac{c \cdot \sin(\alpha)}{b - c \cdot \cos(\alpha)} \right)$$

$$\sin^2(\gamma) \cdot \left(\frac{b}{c} - \cos(\alpha)\right)^2 = \sin^2(\alpha) \cdot \cos^2(\gamma) = \sin^2(\alpha) \cdot (1 - \sin^2(\gamma)) = \sin^2(\alpha) - \sin^2(\alpha) \cdot \sin^2(\gamma)$$

$$\sin^2(\gamma) \cdot \left(\left(\frac{b}{c} - \cos(\alpha)\right)^2 + \sin^2(\alpha)\right) = \sin^2(\alpha)$$

$$\sin^2(\gamma) \cdot \left(\frac{b^2}{c^2} - 2 \cdot \frac{b}{c} \cdot \cos(\alpha) + \cos^2(\alpha) + \sin^2(\alpha)\right) = \sin^2(\alpha)$$

$$\sin^2(\gamma) \cdot \left(\frac{b^2}{c^2} - 2 \cdot \frac{b}{c} \cdot \cos(\alpha) + 1\right) = \sin^2(\alpha)$$

$$\frac{\sin^2(\gamma)}{c^2} \cdot (b^2 - 2 \cdot b \cdot c \cdot \cos(\alpha) + c^2) = \sin^2(\alpha)$$

$$\left(\frac{\sin(\gamma)}{c}\right)^2 = \frac{\sin^2(\alpha)}{b^2 - 2 \cdot b \cdot c \cdot \cos(\alpha) + c^2} \quad = \left(\frac{\sin(\alpha)}{a}\right)^2 = \frac{\sin^2(\alpha)}{a^2}$$ par le théorème du sinus.

En conséquence : $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos(\alpha)$ qui est le théorème du cosinus.